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Dynamics Analysis of Two Storey Plane Rectangular Rigid Frame Subjected to Ground Motion

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Abstract
The dynamic analysis of plane rectangular rigid frame subjected to ground motion is presented. The matrix stiffness method, static condensation method, Szilard household characteristic equation method, Newton-Raphson's method and Gaussian elimination method were used in the analysis. The plane rectangular rigid frame was fixed at both support and the masses were concentrated at the floors level (lump masses). Matrix stiffness method was applied to generate the element stiffness matrix. Various element stiffness matrices were combined to form the global stiffness matrix. Due to large size of global stiffness matrix, the method of static condensation was applied to reduce the size. By substituting the condensed stiffness matrix in the free vibration equation and applying Szilard household characteristic equation method, Newton-Raphson's method and Gaussian elimination method, the eigenvalues and eigenvectors were obtained. From eigenvectors we obtain the base shear forces and base overturning moments. A comprehensive visual basic excel program was developed from this method to obtain eigenvalue, eigenvector, base shear force and base overturning moment as follows: 0.0464, 0.33, 4.526KN and 9.051KNm respectively. Comparing the present study and the direct stiffness method for two storey frame, gave a percentage different of -0.0273 for eigenvalue w_1 and -17.303 for eigenvectorϕ_1.

Keywords: Identical, eigenvalues, overturning, eigenvectors and frames.

Introduction
Structural dynamics is a type of structural analysis which covers the behaviour of structures subjected to dynamic loading. Dynamic loads include earthquake loads, people, windstorm, waves, vibration of the ground due to a blast nearby, hurricanes, flooding, tornadoes, vibration effect from compound disc player, vibrations effect from moving vehicles, vibration effect from heavy construction equipment etc. Any structure can be subjected to dynamic loading. Dynamic analysis can be used to find dynamic displacements, eigenvalue, eigenvector, shear force and bending moment. It has been observed that most of the buildings designed, and constructed in Nigeria by contractors were designed and constructed without considering the effects of these loads. This negligence may have resulted in the cracks or collapse of buildings in our vicinity. Cases of building cracks or collapses at the neighboring town of an area defected by earthquake have been observed. For example, the 2011 Virginia earthquake that cracked and collapsed structures in the neighbouring towns, Ottawa Canada and West Cleveland Ohio (Raghunandan and Liel, 2013).

Nigeria has recorded earth movements in some of the above mentioned areas. In 2012, some parts of Nigeria experienced flooding which may have resulted to the crack or collapse of buildings located at the area. Areas like Aguleri in Anambra State, Ohaji /Egbema in Imo State, Lokoja in Kogi State etc were affected. Some of the buildings at those areas may have collapsed due to the effect of dynamic load caused by the flood. At Borno State there have been several blasts from the terrorist group called Boko Haram. The vibration effects from these blasts have contributed to the cracks and collapse of buildings around these areas. Some of those structures may have been saved if there dynamic loads were considered at the design stage. There are cases where man induces this ground motion on the purpose of

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beautifying the city, for instance building located at areas where road constructions are going on, experiences ground motion due to vibration effect from the compactor and other heavy equipment used in the construction. A case was observed in Michael Okpara University of Agriculture Umudike, Abia State. An ICT building developed several cracks after the reconstruction of the access road that passed across the frontage of the building. In some cases the engineer being fully aware of these earth movements, design the building without considering the dynamic loading, because of the rigorous, time consuming and lack of method that can manually handle the dynamic analysis of large frames without making use of software. It is this rigorous, time consuming and inefficient manual method problems that this work has proffers a simpler and easier way to tackle by developing a method that can manually handle the dynamic analysis of large plane rectangular rigid frames subjected to ground motion and also comprehensive computer program that can handle both the static and vibration analysis.

Chen and Krauthammer used a combined finite element-finite difference method with sub structuring approach to solve the soil-structure interaction problems subjected to seismic loading. The study concentrated on the importance of the mathematical modeling of the interaction system (Chen and Krauthammer, 1989). Mahmood and Ahmed used the interface element between the foundation and soil surface to incorporate the slip and separation modes expected to occur during the period of vibration, and joined two different elements having different degrees of freedom at their nodes. They also paid attention to the: (*a*) actual representation of the frame members by the beam–column elements with the nonlinear behavior of these members according to the axial force–bending moment interaction diagram; and (*b*) accurate modeling of the nonlinear behavior of soil by using the cap model. But it was found that the inclusion of this element affects the dynamic response of the structure due to the relative motions allowed in this case compared with the bonded case (Mahmood and Ahmed, 2007). Phan et al. compared the elastic response of 6-story building subjected to an earthquake loading using two different boundary conditions, in the first, the frame is assumed to be fixed at supports, and in the second case the interaction between the structure and the subsoil is considered. The study clarified the effect of considering the soil-structure interaction on the structural response. The soil medium was represented by simple elastic springs (Phan et al, 1994). Gunay and Mosalam (2010) provide examples of concrete- and shear-dominated failures observed after a recent earthquake. Consequently, there is an urgent need to perform safety assessments to identify and upgrade such structures. Currently available dynamic analysis methods, such as Perform3D (CSI 2006) and SeismoStruct (SeismoStruct 2010), however, typically neglect shear-related effects by default. This omission may result in grossly unconservative and unsafe response predictions. (Guner and Vecchio, 2012). Dunand et al. estimated the modal frequencies and damping ratios of 26 reinforced concrete buildings from ambient vibration records. The estimated data were used in the analysis of these buildings taken into consideration the soil-structure interaction. The main emphasis was to determine the damping resulting from the soil layer itself during the interaction analysis and its contribution in absorbing vibration energy of the structure (Dunand et al, 2003).

If you take a closer look to all the aforementioned work by different scholars, you will notice that there methods were very cumbersome, time consuming, requires a computer programming for large frames and lack a comprehensive program that can analyze the frame from structural to dynamic bending moment. It is these problems that this work will address with a simplified method that can handle large frames manually and a combined comprehensive program that can analyze from static to dynamic bending moments.

Idealization of frame structures

A frame structure can be idealized as an assemblage of elements – beams, columns, walls – interconnected at nodal points or nodes. These nodal points are numbered as well as the element (beams and columns) of the frame. In this idealization, the beams and column systems are rigid (infinitely stiff) in flexure; several factors are neglected: Axial deformation of beams and columns, and the effect of axial forces on the stiffness of the columns. This is because the inertial effect associated with rotations and vertical displacements of the joints are usually not significant. The mass is distributed throught the building, but we will idealize it as concentrated at the floor levels that is lump mass. The frame is fixed at both support.

Elements and global stiffness matrix

$$\begin{Bmatrix} P_i \\ M_i \\ P_j \\ M_i \end{Bmatrix} = EI/L^3 \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} V_i \\ \theta_i \\ V_J \\ \theta_J \end{Bmatrix}$$
……………………………………………………………………………………1

Equation (1) was applied to every elements of the frame to generate the element stiffness matrices. The various element stiffness matrices were combined together to form the global stiffness matrix. See Ozioko, 2014 for details.

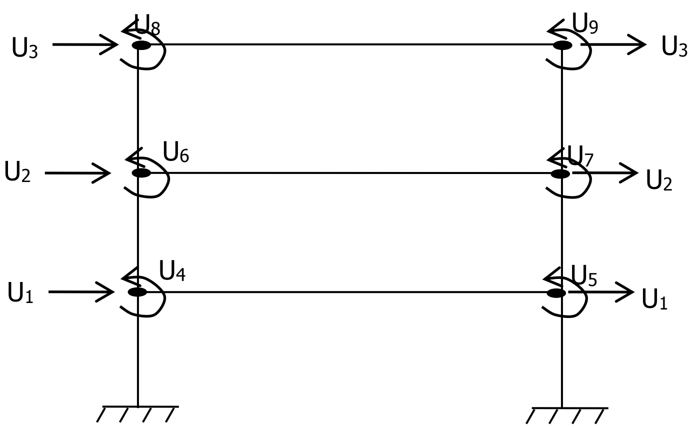

Figure- 1: Three storey frame showing the lateral and rotational displacement.

The method of static condensation

This method is used to systematically eliminate those degrees of freedom that has zero masses assigned to them.

$$\begin{array}{c|c} K_{tt} & K_{t0} \\ \hline K_{0t} & K_{00} \end{array}$$
…………………………………………………………………….2

From equation 2,
k_{tt} = square matrix (txt) where "t" correspond to the number of translational degrees of freedom .
k_{00} = square matrix (0x0) where "0" correspond to the number of rotational degrees of freedom.
k_{t0} = rectangular matrix (0x0) where "t" and "0" are as applied above in k_{tt} and k_{00}.
k_{0t} = rectangular matrix (0xt) where "t" and "0" are as applied above in k_{tt} and k_{00}.

Ozioko, 2014 deduced condensed matrix ($\hat{k}_{tt}$) equation as shown below.
$\hat{k}_{tt} = k_{tt} - k_{0t}^T k_{00}^{-1} k_{0t}$ ……………………………………………………………………3

Free vibration analysis

To determine the eigenvalue, we have to solve for w^2 in equation (7) below. Thus
Det($\hat{k}_{tt}$ - $w^2 m_{tt}$) ………………………………………………………………..4
Where $w^2 = \lambda$ = eigenvalue, m_{tt} = lump mass matrix.
Characteristic equations are formed by solving equation (4) and the derivation and solution to the household characteristic equation (equation 5) are shown in Ozioko, 2014.
$\lambda^n + k_1\lambda^{n-1} + k_2\lambda^{n-2} +$ …….+ $k_{n-1}\lambda + k_n = 0$ ………………………………5
Where coefficients k_1, k_2, …………k_n are defined by
$k_1 = -T_1$ ……………………………………………………………………...6a

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$k_2 = -\frac{1}{2}(k_1T_1 + T_2)$ ………………………………………………………...6b
$k_3 = -\frac{1}{3}(k_2T_1 + k_1T_2 + T_3)$ ………………………………………………...6c
. .
. .
. .
$k_n = -\frac{1}{n}(k_{n-1}T_1 + k_{n-2}T_2 + \ldots\ldots + k_1T_{n-1} + T_n)$ …………………….....6d

In equation 6, T_1, T_2, T_3, …………T_n represent the traces of the matrices A^n, which are defined as the sum of the diagonal elements of the corresponding matrix. See Ozioko, 2014.
The solution to the polynomial equation is given by equation (7) as stated below.

$r_2 = r_1 - \frac{f(r_1)}{f^I(r_1)}$ ………………………………………………………………………..7

Equation (12) above is used to solve the polynomial equation of nth degree by trial and error. Where
r_2 = root of the polynomial equation gotten by solving equation (7)
r_1 = assumed root for trials

f(r_1) = the value obtained when r_1 is substituted in thepolynomial equation.
$f^I(r_1)$ = the value obtained when $r_1$is substituted in the differential of the polynomial equation.

Eigenvector or natural mode *Φ*
The eigenvectors are obtained by solving equation (8) below using Gaussian elimination mehtod.
$(\hat{k}_{tt} - \Lambda_n m_{tt})\Phi_{nj} = 0$ ……………………………………………………………………8

Forced vibration analysis
BASE SHEAR FORCE DUE TO NTH MODE
Chopra 2007 Deduced the base shear force due to nth mode and is given as
$V_{bn} = V_{bn}^{st}A_n$ …………………………………......………………………..........................9
But $V_{bn}^{st} = \sum_{J=1}^{N} S_{jn} = \Gamma_n L_n^h$ ………………………………...…………………………10
Where $\Gamma_n = \frac{L_n^h}{M_n}$ ………………………………..…………………………………………..11
$L_n^h = \sum_{J=1}^{N} m_j \Phi_{jn}$ ………………………………..………………………………12
$M_n = \sum_{J=1}^{N} m_j V_{Jn}^2$ …………………………....……………………………………13
Equation (11), (12) and (13) gives the values of the modal properties.

Overturning Moment Due To Nth Mode
Thus the base moment which is the base overturning moment is given as
$M_{bn} = M_{bn}^{st}A_n = \Gamma_n L_n^{\theta} A_n$ ………………………………………………………………….14 but
$M_{bn}^{st} = \sum_{J=1}^{N} h_j s_{jn} = \Gamma_n L_n^{\theta}$ ……………………………………………………………………15
Where $L_n^{\theta} = \sum_{J=1}^{N} h_j m_j \Phi_{jn}$

Numerical application and discussion
Description of the Problem
The frame structure shown in figure (1), is considered for investigating the structural response predicted by the linear dynamic analysis using the developed computer program of the present study. Details of the beams and columns of the frame are shown in Figure (1). The system is subjected to an identical ground motion. The axial deformation is neglected. The system is assumed to have lump masses and undamped. Flexural rigidity is calculated in terms of EI. The properties of the materials used in the analysis are summarized as follows:
Flexural rigidity = EI(KNM^2) and 2EI(KNM^2)
Mass = M(KG) and 2M(KG) and Height = h(m) and 2h(m)

Dynamic Analysis
The dynamic analysis was carried out, using our method and our program. The results obtained were listed below and also compared with the results of another author as follows:

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Direct Stiffness Method

$w_1 = 2.198\sqrt{EI / m}$, $w_2 = 5.850\sqrt{EI / m}$, $\Phi_1 = \begin{matrix} 0.3871 \\ 1.0 \end{matrix}$ and $\Phi_2 = \begin{matrix} -1.292 \\ 1.0 \end{matrix}$.

Direct Equilibrium Method

$w_1 = 3.464\sqrt{EI / m}$, $w_2 = 6.928\sqrt{EI / m}$, $\Phi_1 = \begin{matrix} 0.5 \\ 1.0 \end{matrix}$ and $\Phi_2 = \begin{matrix} -1.0 \\ 1.0 \end{matrix}$

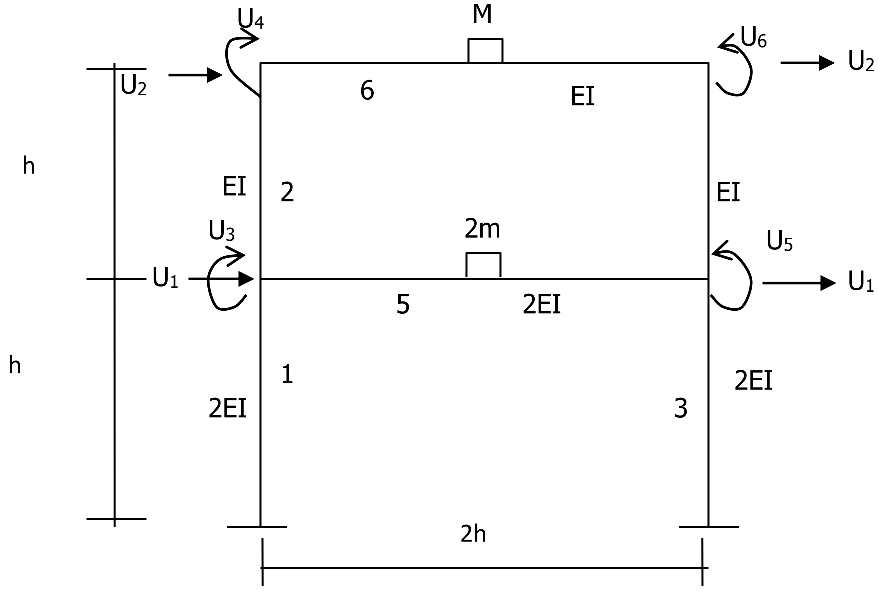

Figure -2: Sample two storey frame that was analyzed.

Present Study

$w_1 = 2.1974\sqrt{EI / m}$, $w_2 = 5.2386\sqrt{EI / m}$, $\Phi_1 = \begin{matrix} 0.33 \\ 1.0 \end{matrix}$ and $\Phi_2 = \begin{matrix} -0.9616 \\ 1.0 \end{matrix}$

The natural frequency or eigenvalue is represented by w_n while the natural mode or eigenvector is represented by Φ_n and $\lambda_n = \sqrt{w_n}$.

Table -1: Comparison of percentage difference of present study over direct stiffness method and direct equilibrium method.

A	B	C	D	E	F	G
(w_1)	2.198	3.464	2.1974	-0.0273	-57.6408	-57.5978
(w_2)	5.850	6.928	5.2386	-11.6711	-32.2491	-12.1808
(Φ_1)	$\begin{matrix} 0.3871 \\ 1.0 \end{matrix}$	$\begin{matrix} 0.5 \\ 1.0 \end{matrix}$	$\begin{matrix} 0.33 \\ 1.0 \end{matrix}$	$\begin{matrix} -17.303 \\ 0 \end{matrix}$	$\begin{matrix} -51.5152 \\ 0 \end{matrix}$	$\begin{matrix} -29.1656 \\ 0 \end{matrix}$
(Φ_2)	$\begin{matrix} -1.292 \\ 1.0 \end{matrix}$	$\begin{matrix} -1.0 \\ 1.0 \end{matrix}$	$\begin{matrix} -0.9616 \\ 1.0 \end{matrix}$	$\begin{matrix} -34.3594 \\ 0 \end{matrix}$	$\begin{matrix} 3.9933 \\ 0 \end{matrix}$	$\begin{matrix} 22.6006 \\ 0 \end{matrix}$

Column A = Parameters to be compared, Column B = Direct stiffness method values in ($\sqrt{EI / mh^3}$), Column C = Direct equilibrium method in ($\sqrt{EI / mh^3}$), Column D = Present study in ($\sqrt{EI / mh^3}$), E = % difference between present study over direct stiffness method, F = % difference between present study over direct equilibrium method, G = % difference between stiffness method over equilibrium method, w_1 and w_2= Eigenvalue 1 and 2 and Φ_1 and Φ_2 = Eigenvector 1 and 2.

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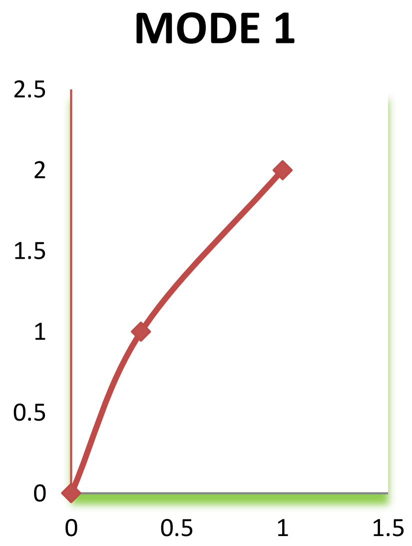

Figure -1a: Graph showing the eigenvectors for the two storey frame

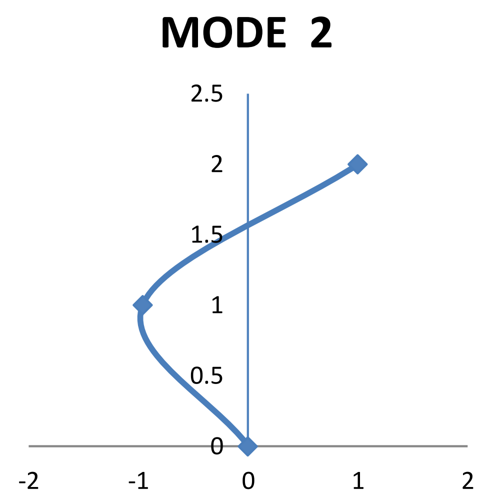

Figure -1b: Graph showing the eigenvectors for the two storey frame

Table -2b: Base shear for the two storey frame in KN

S/N	V_{bbn}
1	4.526
2	0.897

Table -2c: overturning moment for the two storey frame in KNM

S/N	L_n^θ	M_{bn}
1	3.32	9.051
2	-1.85	1.795

From table 1 you will notice that mode 1 has only positive values and from the graph of figure 4a you will also observe that mode 1 is not well distributed. It has its graph drawn on one side of the graph and will need a higher shear force and moment to counterbalance the motion while mode 2 has both positive and negative values. Its graph is better distributed on both sides and has a more uniform motion. Mode 2 will require a lesser moment and shear forceto balance its motion.

Thus these two modes showed the possible movements of the two storey plane rectangular rigid frame when subjected to such ground motion.

A close look at the overturning moment and base shear force will reveal that more uniform motion with more uniform graphs will produced lesser moments while a scattered motion with scattered graphs will produce larger moment because, the effect of the forces are felt more on one side of the structure.

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Conclusions

Based on the results of this research, the following conclusions were drawn;

. Plane rectangular rigid frame subjected to ground motion was analyzed.

. The method developed can manually handle the dynamic analysis of large plane rectangular rigid frames subjected to ground motion.

. The comprehensive user friendly software program developed in this study can efficiently analyze a plane rectangular rigid frame and generate its, condense stiffness matrix, eigenvalues, eigenvectors, base shear force, base moments and plot the eigenvector graphs.

. The results of the analysis of the two storey frame using our method converged with the ones from direct stiffness method and direct equilibrium method used in Chopra (2007).

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